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ATTORNEY'S DOCKET NUMBER

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09/463907

U.S. APPLICATION NO. (If known, see 37 CFR 1.5)

INTERNATIONAL APPLICATION NO.	INTERNATIONAL FILING DATE	PRIORITY DATE CLAIMED
PCT/JP99/02924	1 June 1999	2 June 1998

## TITLE OF INVENTION

APPARATUS AND METHOD FOR EVALUATING RANDOMNESS OF FUNCTIONS, RANDOM  
FUNCTION GENERATING APPARATUS AND METHOD, AND RECORDING MEDIUM HAVING  
RECORDED THEREON PROGRAMS FOR IMPLEMENTING THE METHODS

## APPLICANT(S) FOR DO/EO/US

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Kazuo

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

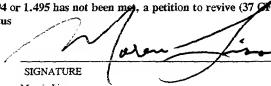
1. ☒ This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. ☐ This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. § 371.
3. ☒ This express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay examination until the expiration of the applicable time limit set in 35 U.S.C. 371(b) and PCT Articles 22 and 39(1).
4. ☐ A proper Demand for International Preliminary Examination was made by the 19th month from the earliest claimed priority date.
5. ☒ A copy of the International Application as filed (35 U.S.C. 371(c)(2))
  - a. ☐ is transmitted herewith (required only if not transmitted by the International Bureau).
  - b. ☒ has been transmitted by the International Bureau.
  - c. ☐ is not required, as the application was filed in the United States Receiving Office (RO/US).
6. ☒ A translation of the International Application into English (35 U.S.C. 371(c)(2)).
7. ☐ Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371(c)(3))
  - a. ☐ are transmitted herewith (required only if not transmitted by the International Bureau).
  - b. ☐ have been transmitted by the International Bureau.
  - c. ☐ have not been made; however, the time limit for making such amendments has NOT expired.
  - d. ☐ have not been made and will not be made.
8. ☐ A translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).
9. ☒ An oath or declaration of the inventor(s) (35 U.S.C. 371(c)(4)).
10. ☐ A translation of the Annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371(c)(5)).

## Items 11. to 16. below concern other document(s) or information included:

11. ☒ An Information Disclosure Statement under 37 CFR 1.97 and 1.98.
12. ☒ An assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.
13. ☒ A **FIRST** preliminary amendment.  
☐ A **SECOND** or **SUBSEQUENT** preliminary amendment.
14. ☐ A substitute specification.
15. ☐ A change of power of attorney and/or address letter
16. ☒ Other items or information:

Copies of forms PCT/IB/308, PCT/IB/301, and PCT/IB/304

09463907-020200

U.S. APPLICATION NO. (if known, see 37 CFR 1.5) <b>097463907</b>		INTERNATIONAL APPLICATION NO. PCT/JP99/02924		ATTORNEY'S DOCKET NUMBER 0162/900547	
<input checked="" type="checkbox"/> The following fees are submitted:				CALCULATIONS	PTO USE ONLY
<b>Basic National Fee (37 CFR 1.492(a)(1)-(5)):</b> Search Report has been prepared by the EPO or JPO.....\$840.00 International preliminary examination fee paid to USPTO (37 CFR 1.482) .....\$670.00 No international preliminary examination fee paid to USPTO (37 CFR 1.482) but international search fee paid to USPTO (37 CFR 1.445(a)(2)).....\$760.00 Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(2)) paid to USPTO.....\$970.00 International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(2)-(4).....\$96.00					
<b>ENTER APPROPRIATE BASIC FEE AMOUNT =</b>				<b>\$840</b>	
Surcharge of \$130.00 for furnishing the oath or declaration later than <input type="checkbox"/> 20 <input type="checkbox"/> 30 months from the earliest claimed priority date (37 CFR 1.492(e)).				\$ 0.00	
Claims	Number Filed	Number Extra	Rate		
Total Claims	52 - 20 =	32	X \$18.00	\$576	
Independent Claims	6 - 3 =	3	X \$78.00	\$234	
Multiple dependent claim(s) (if applicable)			+ \$260.00	\$260	
<b>TOTAL OF ABOVE CALCULATIONS =</b>				<b>\$1,910</b>	
Reduction by 1/2 for filing by small entity, if applicable. Verified Small Entity statement must also be filed. (Note 37 CFR 1.9, 1.27, 1.28)				\$	
<b>SUBTOTAL =</b>				<b>\$1,910</b>	
Processing fee of \$130.00 for furnishing the English translation later than <input type="checkbox"/> 20 <input type="checkbox"/> 30 months from the earliest claimed priority date (37 CFR 1.492(e)).				\$0	
<b>TOTAL NATIONAL FEE =</b>				<b>\$1,910</b>	
Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31). \$40.00 per property +				\$40	
<b>TOTAL FEES ENCLOSED =</b>				<b>\$1,950</b>	
				Amount to be refunded \$	
				charged \$	
a. <input checked="" type="checkbox"/> A check in the amount of \$1,950 to cover the above fees is enclosed. b. <input type="checkbox"/> Please charge my Deposit Account No. <u>22-0185</u> in the amount of \$_____ to cover the above fees. A duplicate copy of this sheet is enclosed. c. <input checked="" type="checkbox"/> The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment to Deposit Account No. <u>22-0185</u> . A duplicate copy of this sheet is enclosed.					
NOTE: Where an appropriate time limit under 37 CFR 1.494 or 1.495 has not been met, a petition to revive (37 CFR 1.137(a) or (b)) must be filed and granted to restore the application to pending status <b>SEND ALL CORRESPONDENCE TO:</b> <b>Pollock, Vande Sande &amp; Amernick, R.L.L.P.</b> 1990 M Street, N.W., Suite 800 Washington, DC 20036-3425 2/2/00					
SIGNATURE  NAME Morris Liss 24,510 REGISTRATION NUMBER					

097463907 020200

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re Application of: :  
: :  
Shiho Moriai et al. :  
: :  
Serial No.: To be assigned : Art Unit: To be assigned  
: :  
Filed: Herewith : Examiner: To be assigned  
: :  
For: APPARATUS AND METHOD : Atty Docket: 0162/00547  
FOR EVALUATING :  
RANDOMNESS OF :  
FUNCTIONS, RANDOM :  
FUNCTION GENERATING :  
APPARATUS AND METHOD, :  
AND RECORDING MEDIUM :  
HAVING RECORDED :  
THEREON PROGRAMS FOR :  
IMPLEMENTING THE :  
METHODS :

**PRELIMINARY AMENDMENT**

Assistant Commissioner for Patents  
Washington, D.C. 20231

Sir:

Prior to examination of the above-identified national phase application, please amend the application as follows:

**IN THE CLAIMS**

Please amend claims 19 and 26 and add new claims 31 and 32 as follows:

At claim 19, line 1, replace "14, or 15" with --14--.

At claim 26, line 1, replace "21, or 22" with --21--.

--31. The random function generating method of claim 15, wherein said candidate functions are each to a composite function composed of at least one function resistant to said differential cryptanalysis and said linear cryptanalysis and at least one function of an algebraic structure different from that of said at least one function.

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--32. The recording medium of claim 22, wherein said candidate functions are each a composite function composed of at least one function resistant to said differential cryptanalysis and said linear cryptanalysis and at least one function of an algebraic structure different from that of said at least one function.

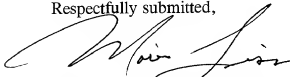
### REMARKS

Claims 1-32 remain in the application. By the foregoing amendment, claims 19 and 26 were amended to eliminate improper multiple dependencies. New claims 31 and 32 are the recitations of claims 19 and 26, respectively, which were eliminated by the foregoing amendment. These changes are not believed to introduce new matter, and entry of this amendment is respectfully requested.

### DEPOSIT ACCOUNT AUTHORIZATION

It is not believed that extensions of time or fees for net addition of claims are required, beyond those which may otherwise be provided for in documents accompanying this paper. However, in the event that additional extensions of time are necessary, then such extensions of time are hereby petitioned under 37 CFR § 1.136(a), and any fees required for consideration of this paper, including fees for net addition of claims, are hereby authorized to be charged to our Deposit Account No. 22-0185.

Respectfully submitted,



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Date: 2/2/00

APPARATUS AND METHOD FOR EVALUATING RANDOMNESS OF  
FUNCTIONS, RANDOM FUNCTION GENERATING APPARATUS AND  
METHOD, AND RECORDING MEDIUM HAVING RECORDED  
THEREON PROGRAMS FOR IMPLEMENTING THE METHODS

5

TECHNICAL FIELD

The present invention relates to an apparatus and method which are applied, for example, to a cryptographic device to evaluate whether candidate functions satisfy several randomness criteria so as to obtain a function that generates an irregular output from the input and hence make the analysis of its operation difficult; the invention also pertains to an apparatus and method for generating a random function evaluated to satisfy the randomness criteria, and a recording medium having recorded thereon programs for implementing these methods.

15

PRIOR ART

Encryption techniques are effective in concealing data. Encryption schemes fall into a secret-key cryptosystem and a public-key cryptosystem. In general, the public-key cryptosystem is more advanced in the research of security proving techniques than secret-key cryptosystem, and hence it can be used with the limit of security in mind. On the other hand, since no security proving techniques have been established for the secret-key cryptosystem, it is necessary to individually deal with cipher attacks when they are found.

25

To construct fast and secure secret-key cryptography, there has been proposed a block cipher scheme that divides data into blocks of a suitable length and enciphers each block. Usually, the block cipher is made secure

by applying a cryptographically not so strong function to the plaintext a plurality of times. The cryptographically not so strong function is called an F-function.

It is customary in the art to use, as an element of the F-function, a  
5 random function, called an S-box, which generates an irregular output from the input thereto, making it difficult to analyze its operation. With the S-box that has the random function capability of providing a unique input/output relationship, it is possible to achieve constant and fast output generation irrespective of the complexity of the random function operation  
10 itself, by constructing the S-box with a ROM that has the input/output relationship as a table. Since the S-box was adopted typically in DES (Data Encryption Standard), its security and design strategy have been studied. Conventionally, the security criterion assumed in the implementation of the S-box is only such that each bit of encrypted data, for  
15 instance, would be a 0 or 1 with a statistical probability of 50 percent—this is insufficient as the theoretical criterion for the security of block ciphers.

In actual fact, cryptanalysis methods for block ciphers that meet the above-mentioned criterion have been proposed: a differential cryptanalysis in literature “E. Biham, A. Shamir, ‘Differential Cryptanalysis of DES-like  
20 cryptosystems,’ Journal of Cryptology, Vol. 4, No. 1, pp.3-72” and a linear cryptanalysis in literature “M. Matsui, ‘Linear Cryptanalysis Method for DES Ciphers,’ Advances in Cryptology-EUROCRYPT’ 93 (Lecture Notes in Computer Science 765), pp.386-397, Springer-Verlag, 1994.” It has been found that many block ciphers can be cryptanalyzed by these methods;  
25 hence, it is now necessary to review the criteria for security.

After the proposal of the differential and linear cryptanalysis methods the block ciphers have been required to be secure against them.

To meet the requirement, there have been proposed, as measures indicating the security against the cryptanalysis methods, a maximum average differential probability and a maximum average linear probability in literature “M. Matsui, ‘New Structure of Block Ciphers with Provable

- 5 Security against Differential and Linear Cryptanalysis,’ D. Gollmann, editor, Fast Software Encryption, Third International Workshop, Cambridge, UK, February 1996, Proceedings, Vol. 1039 of Lecture Notes in Computer Science, pp. 205-218, Springer-Verlag, Berlin, Heidelberg, New York, 1996.” It is indicated that the smaller the measures, the higher the security  
10 against the respective cryptanalysis.

Moreover, it has recently been pointed out that even ciphers secure against the differential and the linear cryptanalysis are cryptanalyzed by other cryptanalysis methods, and consequently, the criterion for security needs a further reappraisal. More specifically, in literature “T. Jackson, L.

- 15 R. Knudsen, ‘The Interpolation Attack on Block Ciphers,’ Fast Software Encryption Workshop (FSE4) (Lecture Notes in Computer Science 1276), pp. 28-40, Springer-Verlag, 1997,” it is described that some ciphers, even if secure against the differential and the linear cryptanalysis, are cryptanalyzed by a higher order differential attack or interpolation attack.

- 20 Other than the higher order differential attack and interpolation attack, a partitioning cryptanalysis generalized from the linear cryptanalysis is introduced in literature “C. Harpes, J. L. Massey, ‘Partitioning Cryptanalysis,’ Fast Software Encryption Workshop (FSE4) (Lecture Notes in Computer Science 1267), pp. 13-27, Springer-Verlag, 1997,” and hence it  
25 is necessary to provide ciphers with sufficient security against this cryptanalysis.

The technology to ensure the security against the differential and the

linear cryptanalysis has been established for the construction of some block ciphers, while as of this point in time no technology has been established yet which guarantees perfect security against the higher order differential attack, the interpolation attack and the partitioning attack. In other words, there

- 5 have not been clarified necessary and sufficient conditions that random functions, i.e. the so-called S-boxes, need to satisfy so as to make ciphers invulnerable to these attacks.

- 10 In designing the S-boxes it is an important issue to provide sufficient security against these attacks. The attacks on the S-boxes utilize any imbalances in their input/output relationships. Accordingly, to design an S-box resistant to an attack is to design an S-box that has little unbalanced, that is, random input/output relationship. Hence, to evaluate the resistance of the S-box to an attack is equivalent to the evaluation of its randomness.

- 15 It is therefore an object of the present invention to provide a function randomness evaluating apparatus and method which find out a criterion closely related to the level of security against each of the above-mentioned attacks, the criterion representing a necessary condition to be met for providing the resistance to the attack (not a necessary and sufficient condition for guaranteeing the security against the attack), and evaluate the  
20 randomness of the function concerned according to the criterion, and a recording medium having recorded thereon the method as a program. Another object of the present invention is to provide an apparatus and method for generating a random function that satisfies the security criterion, and a recording medium having recorded thereon the method as a program.

25

## DISCLOSURE OF THE INVENTION

The function randomness evaluating apparatus and method



according to the present invention execute at least one of the processes of:

calculating the minimum value of the degree of a Boolean polynomial regarding the input by which each output bit of the function to be evaluated is expressed, and evaluating the resistance of the function to

5 higher order differential cryptanalysis accordingly;

when fixing a key  $y$  and letting  $x$  denote the input, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over the Galois field which is composed of elements equal to a prime  $p$  or a power of the prime  $p$ , then calculating the number of terms of the polynomial, and evaluating the

10 resistance of the function to interpolation cryptanalysis accordingly;

dividing all inputs of the function to be evaluated and the corresponding outputs into input subsets and output subsets, then calculating an imbalance of the relationship between the subset of an input and the subset of the corresponding output with respect to their average

15 corresponding relationship, and evaluating the resistance of the function to partitioning cryptanalysis accordingly; and

calculating, for every set of input difference  $\Delta x$  and output mask value  $\Gamma y$  of the function  $S(x)$  to be evaluated, the number of inputs  $x$  for which the inner product of  $(S(x)+S(x \Delta x))$  and the output mask value  $\Gamma y$  is

20 1, and evaluating the resistance of the function to differential-linear cryptanalysis accordingly .

The random function generating apparatus and method according to the present invention generate candidate functions each formed by a plurality of functions of different algebraic structures and having a plurality

25 of parameters, evaluates the resistance of each candidate function to cryptanalysis, and select candidate functions of higher resistance to the cryptanalysis.

## BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 is a block diagram illustrating an example of the functional configuration of each of the random function generating apparatus and the function randomness evaluating apparatus according to the present invention.

Fig. 2 is a block diagram depicting an example of the basic configuration of the random function generating apparatus according to the present invention.

Fig. 3 is a flowchart showing an example of a procedure of an embodiment of the random function generating apparatus according to the present invention.

## BEST MODE FOF CARRYING OUT THE INVENTION

Embodiment according to a first aspect of the present invention

In Fig. 1 there is depicted the functional configuration of an embodiment of each of the random function generating apparatus and the function randomness evaluating apparatus according to the present invention. An input part 11 inputs therethrough data and a parameter that are needed to generate a candidate function in a candidate function generating part 12. The candidate function generating part 2 generates a candidate function based on the input provided through the input part 11, and provides its parameter value, the input value and the calculation result (an output value) to a storage part 13. Various pieces of data thus stored in the storage part 13 are read out therefrom and fed to a differential-cryptanalysis resistance evaluating part 14a, a linear-cryptanalysis resistance evaluating part 14b, a higher-order-differential-attack resistance evaluating part 14c, an interpolation-attack resistance evaluating part 14d, a partitioning-attack

resistance evaluating part 14e, a differential-linear-attack resistance evaluating part 14f, and a criteria evaluating part 14g for evaluating other criteria. Based on the results of evaluations made in the respective evaluating parts, candidate functions of high resistance to the attacks are  
5 selected in a function select part 15 and stored in a storage part 16, from which a required one of the candidate functions is read out and provided to the outside via an output part 17.

In the function randomness evaluating apparatus according to the present invention, the functions to be evaluated are provided via the input  
10 part 11 to the respective evaluating parts 14a to 14g for the evaluation of their randomness.

A description will be given below of security criteria for the differential cryptanalysis, the linear cryptanalysis, the higher order differential attack, the interpolation attack, the partitioning attack and the  
15 differential-linear attack and of necessary conditions for the security criteria to have resistance to the respective attacks. In the following description, let  $n$  and  $m$  be arbitrary natural numbers and consider, as the S-box (a random function), a function  $S$  of an  $n$ -bit input and an  $m$ -bit output:  $GF(2)^n \rightarrow GF(2)^m$ .  $GF(2)^n$  represents a set of all  $n$ -bit data.

20 (a) Necessary Condition for Resistance to Differential Cryptanalysis

A description will be given below of a criterion for differential cryptanalysis is defined as a measure of the resistance thereto of the S-box, a method for measuring the criterion and a necessary condition for the resistance to differential cryptanalysis. In the differential cryptanalysis  
25 method, an observation is made of the difference between outputs (an output difference value) of the S-box corresponding to the difference between its two inputs (an input difference value), and if a large imbalance is found

between them, it can be used to cryptanalyze the whole cipher.

Letting the input to the S-box be represented by  $x$ , the difference value between the two inputs by  $\Delta x$ , the difference value of the two outputs corresponding to the two inputs by  $\Delta y$ , the function of the S-box by  $S$  and the output  $y$  from the S-box for the input thereto by  $y=S(x)$ , let  $\delta_s(\Delta x, \Delta y)$  be the number of those  $x$ s of all  $n$ -bit inputs  $x$  which satisfy the following equation (1) for an arbitrary input difference value  $\Delta x$  and an arbitrary output difference value  $\Delta y$ .

$$S(x) + S(x + \Delta x) = \Delta y \quad (1)$$

where "+" is usually defined by the exclusive OR (XOR) for each bit. As described in literature "X. Lai, J. M. Massey, and S. Murphy, 'Markov Ciphers and Differential Cryptanalysis,' In D. W. Davies, editor, *Advances in Cryptology-EUROCRYPT '91*, Volume 547 of Lecture Notes in Computer Science, pp. 17-38, Springer-Verlag, Berlin, Heidelberg, New York, 1991," the difference operation can be substituted with an arbitrary binary operation that provides a general inverse; the differential cryptanalysis method mentioned herein includes them. The differential cryptanalysis utilizes an imbalance in the relationship between the operation results on two arbitrary inputs and the two outputs corresponding thereto.

The number  $\delta_s(\Delta x, \Delta y)$  of inputs  $x$  that satisfy Eq. 81) for a given pair of  $\Delta x$  and  $\Delta y$  is expressed by the following equation (2):

$$\delta_s(\Delta x, \Delta y) = \#\{x \in GF(2)^n \mid S(x) + S(x + \Delta x) = \Delta y\} \quad (2)$$

where  $\#\{x \mid \text{conditional equation}\}$  represents the number of inputs  $x$  that satisfy the conditional equation. The number  $\delta_s(\Delta x, \Delta y)$  of inputs  $x$  can be calculated from Eq. (2) for all pieces of  $n$ -bit data  $\Delta x$  as the input difference value, except 0, and all pieces of  $m$ -bit data  $\Delta y$  as the output difference value. A combination of  $\Delta x$  and  $\Delta y$  that maximizes the above-said

number constitutes a vulnerability to the differential cryptanalysis--this means that the smaller the maximum value of  $\delta_s(\Delta x, \Delta y)$ , the higher the resistance to differential cryptanalysis. Therefore, it is the necessary condition for the resistance to differential cryptanalysis that the criterion for differential cryptanalysis,  $\Delta_s$ , given by the following equation (3) is small.

$$\Delta_s = \max \delta_s(\Delta x, \Delta y) \quad (3)$$

Eq. (3) indicates selecting that one of all the combinations of  $\Delta x \neq 0$  and  $\Delta y$  which provides the maximum value of  $\delta_s$  and using it as the value of  $\Delta_s$ .

#### (b) Necessary Condition for Resistance to Linear Cryptanalysis

A description will be given below of the definition of a criterion for linear cryptanalysis as a measure of the resistance thereto of the S-box, a method for measuring the criterion and a necessary condition for the resistance to linear cryptanalysis.

In the linear cryptanalysis method, an observation is made of an arbitrary exclusive OR between the input and output values of the S-box, and if a large imbalance is found between them, it can be used to cryptanalyze the whole cipher.

Letting the input to the S-box be represented by  $x$ , an input mask value by  $\Gamma x$  and an output mask value by  $\Gamma y$ ,  $\lambda_s(\Gamma x, \Gamma y)$  defined by the following equation (4) can be calculated for a certain input mask value  $\Gamma x$  and a certain output mask value  $\Gamma y$ .

$$\lambda_s(\Gamma x, \Gamma y) = \left| 2^{x\#} \left\{ x \in (2)^n \mid x \cdot \Gamma x = S(x) \cdot \Gamma y \right\} - 2^n \right| \quad (4)$$

where “ $\cdot$ ” is usually defined by the inner product.  $x \cdot \Gamma x$  means summing-up of all only those bit values in the input  $x$  which correspond to “1s” in the mask value  $\Gamma x$ , ignoring the bit values corresponding to “0s”. That is,  $x \cdot \Gamma x = \sum x_i$  (where  $\sum$  is the sum total of  $i$ -th bits in  $\Gamma x$  which are “1s”),

where  $x=(x_{n-1}, \dots, x_0)$ . The same is true of  $y \cdot \Gamma y$ . Accordingly, Eq. (4) expresses the absolute value of a value obtained by subtracting  $2^n$  from the double of the number of those of all  $(2^n)$  n-bit inputs  $x$  which satisfy  $x \cdot \Gamma x = S(x) \cdot \Gamma y$  for given sets of mask values  $(\Gamma x, \Gamma y)$ .

- 5 From Eq. (4)  $\lambda_s(\Gamma x, \Gamma y)$  can be calculated for all sets of n-bit data  $\Gamma x$  as the input mask value and m-bit data  $\Gamma y$  as the output mask value, except 0. A combination of  $\Gamma x$  and  $\Gamma y$  that maximizes  $\lambda_s(\Gamma x, \Gamma y)$  constitutes a vulnerability to the linear cryptanalysis-- this means that the smaller the maximum value of  $\lambda_s(\Gamma x, \Gamma y)$ , the higher the resistance to linear cryptanalysis. Therefore, it is the necessary condition for the resistance to linear cryptanalysis that the criterion for linear cryptanalysis,  $\Lambda_s$ , given by the following equation (5) is small.

$$\Lambda_s = \max \lambda_s(\Gamma x, \Gamma y) \quad (5)$$

- Eq. (5) indicates selecting that one of all the combinations of  $\Gamma x$  and  $\Gamma y \neq 0$  which provides the maximum value of  $\lambda_s(\Gamma x, \Gamma y)$  and setting as the value of  $\Lambda_s$ .

- (c) Necessary Condition for Resistance to Higher Order Differential Attack
- A description will be given below of the definition of a criterion for higher order differential attack as a measure of the resistance thereto of the S-box, a method for measuring the criterion and a necessary condition for the resistance to higher order differential attack.

- The higher order differential attack utilizes the fact that the computation of a higher order differential the intermediate output in the course of encryption with respect to the input provides a key-independent constant. An arbitrary bit of arbitrary intermediate data during encryption can be expressed by a Boolean polynomial regarding the input. For instance, a bit  $y_i$  of certain intermediate data can be expressed by a Boolean

polynomial regarding an N-bit input x as follows:

$$y_j = x_0 + x_1 x_3 + x_0 x_2 x_3 + \dots + x_1 x_4 x_5 x_6 \dots x_N \quad (6)$$

When the degree of the Boolean polynomial is d, the calculation of the (d+1)-th order differential (for instance, XOR of  $2^{d+1}$  outputs) results in providing a key-independent constant; attacks on ciphers of low-degree Boolean polynomials are reported in the afore-mentioned literature A.

With a low-degree Boolean polynomial representation of an F-function, an insufficient number of iterations of the F-function will not raise the degree of the Boolean representation of the whole cipher, increasing the risk of the cipher being cryptanalyzed. Hence, a necessary condition for making the cipher secure against higher order differential attack without increasing the number of iterations of the F-function is that the degree of the Boolean polynomial representation of the S-box as a component of the F-function is also high.

15 For

$$\text{S-box } S: \text{GF}(2)^n \rightarrow \text{GF}(2)^m; x \mapsto S(x),$$

set

$$y = S(x), \quad (7a)$$

$$x = (x_{n-1}, x_{n-2}, \dots, x_0) \in \text{GF}(2)^n, \quad (7b)$$

$$20 \quad y = (y_{m-1}, y_{m-2}, \dots, y_0) \in \text{GF}(2)^m \quad (7c)$$

And a set of variables  $X = \{x_{n-1}, x_{n-2}, \dots, x_0\}$  is defined. At this time, a Boolean function  $y_i = S_i(x)$  is defined as follows:

$$S_i: \text{GF}(2)^n \rightarrow \text{GF}(2); x \mapsto S_i(x) \quad (8)$$

Let  $\deg_x S_i$  denote the degree of the Boolean function  $S_i$  ( $0 \leq i \leq m-1$ )

25 regarding the variable set X. Let the minimum value of  $\deg_x S_i$  ( $0 \leq i \leq m-1$ ) be represented by  $\deg_x S$ , which is the criterion for higher order differential attack.

$$\deg_x S = \min(\deg_x S_i) \quad (9)$$

where min is conditioned by  $0 \leq i \leq m-1$ .

A necessary condition that the S-box needs to satisfy to provide security against higher order differential attack is that  $\deg_x S$  has a large value. It is known that when S is bijective (i.e. the input/output relationship can be determined uniquely in both directions), the maximum value of  $\deg_x S$  is  $n-1$ .

#### (d) Necessary Condition for Resistance to Interpolation Attack

A description will be given below of the definition of a criterion for interpolation attack as a measure of the resistance thereto of the S-box, a method for measuring the criterion and a necessary condition for the resistance to interpolation attack.

The principle of interpolation attack is as follows: With a key k fixed, a ciphertext y can be expressed, for example, by the following equation using a polynomial  $f_k(x)$  over  $GF(q)$  regarding a plaintext x.

$$y = f_k(x) = c_{q-1}x^{q-1} + c_{q-2}x^{q-2} + \dots + c_jx^j + \dots + c_1x^1 + c_0x^0 \quad (10)$$

where q is a prime or its power. When the number of terms of non-zero coefficients contained in the polynomial  $f_k(x)$  with respect to x is c, the polynomial  $f_k(x)$  can be constructed as by the Lagrange interpolation theorem if c different sets of plaintexts and the corresponding sets of ciphertexts  $(x_i, y_i)$  (where  $i = 1, \dots, c$ ) are given. By this, a ciphertext corresponding a desired plaintext x can be obtained.

The larger the number of terms contained in the polynomial  $f_k(x)$ , the larger the number of sets of plaintexts and ciphertexts necessary for interpolation attack using the polynomial representation  $f_k(x)$  over  $GF(q)$ , and the attack becomes difficult accordingly or becomes impossible.

When the number of terms contained in the polynomial representation over  $GF(q)$  of the S-box is small, there is a possibility that the



number of terms contained in the polynomial of the whole cipher over  $GF(q)$  decreases. Of course, even if the number of terms contained in the polynomial over  $GF(q)$  of the S-box is large, care should be taken in the construction of the whole cipher to avoid that the terms cancel out each other, resulting in a decrease in the number of terms contained in the polynomial over  $GF(q)$  of the whole cipher; however, this concerns encryption technology, and as the criterion of the S-box for interpolation attack, it is a necessary condition for the resistance to interpolation attack that the number of terms contained in the polynomial representation over  $GF(q)$  is large. Let the number of terms contained in the polynomial representation over  $GF(q)$  of the function  $S$  of the S-box be represented by  $\text{coeff}_q S$ , which is used as the criterion for interpolation attack using the polynomial representation over  $GF(q)$ .

Since the interpolation attack exists by the number of possible  $qs$ , it is desirable to calculate the number of terms  $\text{coeff}_q S$  in as many polynomials over  $GF(q)$  as possible and make sure that they do not take small values.

(e) Necessary Condition for Resistance to Partitioning Cryptanalysis

A description will be given below of the definition of a criterion for partitioning cryptanalysis as a measure of the resistance thereto of the S-box, a method for measuring the criterion and a necessary condition for providing the resistance to partitioning cryptanalysis. In partitioning cryptanalysis, an observation is made of some measure which holds for a certain subset of the whole plaintext set and a certain subset of the whole ciphertext set, and if a large imbalance is found in the measure, then it can be used to cryptanalyze the whole cipher. As "some measure  $I$ " there are mentioned a peak imbalance and a squared Euclidean imbalance in literature "C. Harpes, J. L. Massey, 'Partitioning Cryptanalysis,' Fast Software Encryption

Workshop (FSE4) (Lecture Notes in Computer Science 1267), pp. 13-27, Springer-Verlag, 1997.”

- In literature “Takeshi Hamada, Takafumi Yokoyama, Tohru Shimada, Toshinobu Kaneko, ‘On partitioning cryptanalysis of DES,’ Proc. in 1998 Symposium on Cryptography and Information Security (SCIS’98-2.2.A),” it is reported that an attack on the whole cipher succeeded through utilization of imbalance observed in input and output sets of the S-box of the DES cipher--this indicates that the criterion for partitioning cryptanalysis similarly defined for the input and output sets of the S-box is a necessary condition for the whole cipher to be secure against partitioning cryptanalysis.

- Let  $u$  divided subsets of the whole set of S-box inputs be represented by  $F_0, F_1, \dots, F_{u-1}$  and  $v$  divided subsets of the whole set of S-box outputs by  $G_0, G_1, \dots, G_{v-1}$ . Suppose that all the subsets contain an equal number of elements. A function  $f$  for mapping the input  $x$  on the subscript  $\{0, 1, \dots, u-1\}$  of each subset will hereinafter be referred to as an input partitioning function and a function  $g$  for mapping the output  $y$  on the subscript  $\{0, 1, \dots, v-1\}$  of each subset as an output partitioning function. That is, the function  $f$  indicates the input subset to which the input  $x$  belongs, and the function  $g$  indicates the output subset to which the output  $y$  belongs. Let partitions  $F$  and  $G$  be defined by

$$F = \{F_0, F_1, \dots, F_{u-1}\},$$

$$G = \{G_0, G_1, \dots, G_{v-1}\}.$$

- Then, an imbalance  $I_s(F, G)$  of an S-box partition pair  $(F, G)$  is given by the following equation (11).

$$I_s(F, G) = \frac{1}{u} \sum_{i=0}^{u-1} I(g(S(x)) | F(x) = i) \quad (11)$$

Expressing  $I(g(S(x))|F(x)=i)$  on the right-hand side of Eq. (11) by  $I(V)$ , this is the afore-mentioned "Measure I." According to the afore-mentioned literature by C. Harpes et al., in the case of using the peak imbalance as this measure, it is expressed as follows:

$$I_P(V) = \frac{v}{v-1} \left( \max_{0 \leq j < v} P[V = j] - \frac{1}{v} \right) \quad (12)$$

In the case of using the squared Euclidean imbalance as this measure, it is expressed as follows:

$$I_E(V) = \frac{v}{v-1} \sum_{j=0}^{v-1} \left( P[V = j] - \frac{1}{v} \right)^2 \quad (13)$$

- $P[V=j]$  represents the probability that the whole output  $y$  corresponding to the whole input  $x$  of an  $i$ -th ( $i=0, \dots, u-1$ ) input group  $F_i$  assigned to an output group  $G_j$  ( $j=0, \dots, v-1$ ), and the sum total of probabilities of the assignment to respective output groups is 1. For example, if  $k_j$  outputs of the whole output  $y$  ( $k_i$  outputs) corresponding to the whole input  $x$  (assumed to consist of  $k_i$  inputs) are assigned to the group  $G_j$ , then the probability of assignment to the group  $G_j$  is  $k_j/k_i$ . The peak imbalance  $I_P(V)$  of Eq. (12) represents a value obtained by normalizing imbalance of the maximum assignment probability relative to an average probability, and the Euclidean imbalance  $I_E(V)$  of Eq. (13) represents a value obtained by normalizing a square-sum of imbalance of the assignment probability from the average one.
- This measure  $I_P$  or  $I_E$  is applied to Eq. (11) to calculate the imbalance  $I_S(F, G)$  for each partition-pair  $(F, G)$ . The partition-pair varies, for instance, with the way of selecting the partitioning functions  $f$  and  $g$ . For example, the division numbers  $u$  and  $v$  are also parameters that are specified by the functions  $f$  and  $g$ .

- The measure given by Eq. (11) is the criterion for the partitioning

attack on the S-box and takes a value greater than 0 and smaller than 1; it is a necessary condition for the resistance to the partitioning attack that the difference between the above value and its one-half is small. Accordingly, the S-box function is chosen which minimizes the value  $|I_s(F, G) - 1/2|$ .

# 5 (f) Necessary Condition for Resistance to Differential-Linear Cryptanalysis

A description will be given below of the definition of a criterion for differential-linear cryptanalysis as a measure of the resistance thereto of the S-box, a method for measuring the criterion and a necessary condition for providing the resistance to differential-linear cryptanalysis.

10 In differential-linear cryptanalysis, an observation is made of, for example, the exclusive OR of S-box input and output difference values, and if a large imbalance is found, then it can be used to cryptanalyze the whole cipher.

15 Letting the S-box input, input difference value and output mask value be represented by  $x$ ,  $\Delta x$  and  $\Gamma y$ , respectively,  $\xi_s(\Delta x, \Gamma y)$  defined by the following equation can be calculated.

$$\xi_s(\Delta x, \Gamma y) = |2\#\{x \in GF(2)^n | [S(x) + S(x + \Delta x)] \cdot \Gamma y = 1\} - 2^n| \quad (14)$$

20 where the operations “+” and “•” are the same as those used in the criterion for differential cryptanalysis and the criterion for linear cryptanalysis, respectively. The maximum value  $\Xi_s$  given by the following equation for any one of all combinations of  $\Delta x$  and  $\Gamma y$  in the measure  $\xi_s(\Delta x, \Gamma y)$  thus calculated is used as the criterion for differential-linear cryptanalysis.

$$\Xi_s = \max_{\Delta x \neq 0, \Gamma y \neq 0} \xi_s(\Delta x, \Gamma y) \quad (15)$$

25 Since a large value of the criterion  $\Xi_s$  may be a weakness in differential-linear cryptanalysis, it is a necessary condition for the resistance thereto that

this value is small (no marked imbalance).

Incidentally, such functions  $S$  as expressed by the following equations are used in some ciphers.

$$S: \text{GF}(2)^n \rightarrow \text{GF}(2)^n: x \rightarrow x^{2^k} \text{ in } \text{GF}(2^n) \quad (16a)$$

$$5 \quad S: \text{GF}(2)^n \rightarrow \text{GF}(2)^n: x \rightarrow x^{2^{k+1}} \text{ in } \text{GF}(2^n) \quad (16a)$$

Letting  $k$  denote a natural number equal to or greater than  $n$ , the differential-linear attack criteria of these functions  $S$  take  $2n$  (the maximum theoretical value). No report has been made of an example in which this property leads to a concrete cipher attack, but it is desirable that the criteria take a small a value as possible.

Next, a description will be given of an embodiment according to a second aspect of the present invention.

The resistance of the  $S$ -box to various attacks is evaluated as described above, but the generation of a highly resistant random function gives rise to an issue of how to select a group of candidate functions. The reason for this is that much complexity is needed to select functions satisfying the above-mentioned condition from an enormous number of functions.

By the way, from an example cited in literature "T. Jakobsen, L. R. Knudsen, 'The Interpolation Attack on Block Cipher,' Fast Software Encryption Workshop (FSE4) (Lecture Notes in Computer Science 1267), pp. 28-40, Springer-Verlag, 1997," it is known that the block cipher is readily cryptanalyzed by the higher order and the interpolation cryptanalysis in the case where the  $S$ -box is formed by a function of a certain algebraic structure selected as a function resistant to the differential and the linear cryptanalysis and the whole cipher is constructed in combination with only by an operation which does not destroy the algebraic structure. On the

other hand, the inventors of this application have found that a composite function, which is a combination of a function resistant to the differential and the linear cryptanalysis with a function of a different algebraic structure (basic operation structure), is also resistant to other attacks in many cases.

5       According to the second aspect of the present invention, functions resistant to the differential and the linear cryptanalysis and functions which have algebraic structures different from those of the first-mentioned functions are combined (composition of functions, for instance) and such composite functions are selected as groups of candidate functions; the  
10       resistance to each cryptanalysis is evaluated for each function group and functions of high resistance are chosen.

Incidentally, the way of selecting the candidate function groups in the present invention is not limited specifically to the above.

15       According to the second aspect of the present invention, a function (for example, a composite function), which is a combination of at least one function resistant to the differential and the linear cryptanalysis with at least one function of a different algebraic structure is selected as a candidate function group. With this scheme, it is possible to efficiently narrow down  
20       from a small number of candidates those functions which are resistant not only to the differential and the linear cryptanalysis but also to attacks which utilize the algebraic structures of the functions used, such as the higher order differential and the interpolation attack.

In the following embodiment according to the second aspect of the present invention, a description will be given of how to design an 8-bit I/O  
25       S-box.

Now, consider that a P-function part 21 for generating a function  $P(x, e)$  and an A-function part 22 for generating a function  $A(y, a, b)$  of an

algebraic structure different from the function  $P(x, e)$  are combined as a candidate function for forming and S-box 20 as shown in Fig. 2.

$$S: GF(2)^8 \rightarrow GF(2)^8; \quad x \mapsto A((P(x, e)), a, b)$$

where

$$P(x, e) = x^e \text{ in } GF(2^8) \quad (17)$$

$$A(y, a, b) = ay + b \pmod{2^8} \quad (18)$$

The function  $P(x, e)$  defined by Eq. (17) is a power function to be defined over Galois Field  $GF(2^8)$ ; this function is resistant to differential cryptanalysis and linear cryptanalysis when the parameter  $e$  is selected suitably, but it has no resistance to higher order differential, linear-differential, interpolation and partitioning attacks. On the other hand, the function  $A(y, a, b)$  defined by Eq. (18) is constructed by a simple addition and a simple multiplication, and this function has no resistance to any of the attacks.

Here, the parameters  $a$ ,  $b$  and  $e$  can freely be set to any natural numbers in the range of from 0 to 255 (i.e.  $2^8-1$ ). Of them, the parameters  $a$  and  $b$  need to have a Hamming weight greater than 3 but smaller than 5, that is, these parameters  $a$  and  $b$  are each 8-bit and required to have three to five "1" (or "0") bits, and the S-box needs to be bijective; the parameters  $a$ ,  $b$  and  $c$  are narrowed down by evaluating whether they satisfy such necessary conditions for providing security against differential cryptanalysis, linear cryptanalysis, interpolation attack and partitioning attack.

In Fig. 3 there is depicted the procedure of an embodiment of the apparatus according to the present invention. Incidentally, the invention is not limited specifically to this embodiment. There is flexibility in the way of selecting functions as candidates for the S-box. Further, the number of design criteria for the S-box is also large, and their priority their priority and

the order of narrowing down the candidates are also highly flexible.

Step S1: In the input part 11, predetermine the range of each of the parameters  $a$ ,  $b$  and  $e$  in Eqs. (17) and (18) to be greater than 0 but smaller than  $2^8-1$ , and limit the Hamming weights  $W_h(a)$  and  $W_h(b)$  of the

5 parameters  $a$  and  $b$  to the range of from 3 to 5.

Step S2: Evaluate whether candidate functions  $S$  are bijective or not. When the parameter  $a$  is an odd number and the parameter  $e$  is prime relative to  $2^8-1$  (which parameter is expressed by  $(e, 255)=1$ ), the functions  $S$  are bijective; select those of the parameters which satisfy these conditions,

10 and discard candidates which do not satisfy them. This processing is performed in the criteria evaluating part 14g in Fig. 1. Alternatively, the parameter  $a$  is obtained by inputting only an odd number in the input part 11.

Step S3: It is known that the Hamming weight  $W_h(e)$  of the parameter  $e$  (which weight indicates the number of "1s" in  $e$  in the binary representation; for example, if  $e = 11101011$ ,  $W_h(e) = 6$ ) and the degree  $\deg_x P$  of the function  $P$  in the Boolean function representation are equal to each other. The, in order to satisfy the condition for a criterion  $\deg_x S$  of the remaining candidate functions  $S$  for higher order differential attack, select those of the remaining candidate functions  $S$  whose parameters  $e$  having a

15 Hamming weights  $W_h(e)$  of 7 that is the theoretically maximum value of  $e$ , that is, select  $e = 127, 191, 223, 239, 251, 253$  and  $254$ . Discard the

20 candidates that do not meet the condition.

Step S4: Determine if any candidates still remain undiscarded.

Step S5: When it is determined in the preceding step that no

25 candidate has survived, ease the condition  $W_h(e) \leftarrow W_h(e)-1$  and then go back to step S3.

Step S6: From the candidate functions remaining after the process of



Step S3, select those candidates for which the criterion  $\Delta_S$  for differential attack defined by Eq. (3) is smaller than a predetermined reference value  $\Delta_R$ . Discard the candidates that do not meet this condition.

Step S7: Determine if any candidates still remain undiscarded after

5 Step S6.

Step S8: If no candidate remains, add a predetermined step width  $\Delta_d$  to the reference value  $\Delta_R$  (ease the condition) to update it, and return to step S6 to repeat the processing.

Step S9: From the candidate functions S remaining after Step S6,  
10 select those candidates for which the criterion  $\Delta_S$  for linear attack defined by Eq. (5) is smaller than a predetermined reference value  $\Delta_R$ . Discard the candidates that do not meet this condition.

Step S10: Determine if any candidates still remain undiscarded after Step S9.

15 Step 11: If no candidate remains, add a predetermined step width  $\Delta_d$  to the reference value  $\Delta_R$  (ease the condition) to update it, and return to step S9 to repeat the processing.

Step S12: From the candidate functions S remaining after step S9, select those candidates for which the criterion  $\Xi_S$  for differential-linear  
20 attack defined by Eq. (15) is smaller than a predetermined reference value  $\Xi_R$ . Discard the candidates that do not meet this condition.

Step S13: Determine if any candidates still remain undiscarded after Step S12.

Step S14: If no candidate remains, add a predetermined step width  
25  $\Xi_d$  to the reference value  $\Xi_R$  (ease the condition) to update it, and return to step S12 to repeat the processing.

As a result, the parameters are narrowed down to those given below.

$(a, b) = (97, 97), (97, 225), (225, 97), (225, 225)$

$e = 127, 191, 23, 239, 247, 251, 253, 254$

Step S15: For the candidate functions  $S$  by all combinations of the parameters remaining after Step S12, calculate the criterion  $I_S(F, G)$  for partitioning attack and select those candidates for which  $|IS(F, G)-1/2|$  is smaller than a reference value  $I_R$ . Discard the candidates that do not meet this condition.

Step S16: Determine if any candidates still remain undiscarded after Step S15.

10 Step S17: If no candidate remains, add a predetermined step width  $I_d$  to the reference value  $I_R$  (ease the condition) to update it, and return to step S15 to repeat the processing.

15 Step S18: For the candidate functions  $S$  by all combinations of the parameters remaining after Step S15, select those candidates for which the criterion  $\text{coeff}_q S$  (where  $q = 28$ ) for interpolation attack, which utilizes the polynomial over  $GF(2^8)$ , is larger than a reference value  $c_{qR}$ , and discard the other candidates.

Step S19: Determine if any candidates still remain undiscarded after Step S18.

20 Step S20: If no candidate remains, subtract a predetermined step width  $c_{qd}$  from the reference value  $c_{qR}$  (ease the condition) to update it, and return to step S19 to repeat the processing.

25 Step S21: From all primes  $p$  in the range of from  $2^8+1$  to  $2^9$ , select those of the candidate functions  $S$  for which the criterion  $\text{coeff}_p S$  for interpolation attack is larger than the reference value  $c_{pR}$ , and discard the other candidates.

Step S22: Determine if any candidates still remain undiscarded after

Step S21.

Step S23: If no candidate remains, subtract a predetermined step width  $c_{pd}$  from the reference value  $c_{pR}$  (ease the condition) to update it, and return to step S21 to repeat the processing.

5 As the result of the evaluation described above, the following combinations (a total of 32 combinations) of parameters are left undiscarded.

(a, b) = (97, 97), (97, 225), (225, 97), (225, 225)

(e) = 127, 191, 223, 239, 247, 251, 253, 254

10 This is identical with the results obtained in step S12. This means that functions secure against every attack taken into account in this embodiment are already obtained in step S12.

Since the 32 functions thus selected are equally strong on the above-mentioned criteria, any of the functions can be used as the S-box.

15 In the evaluation of the S-box or in the function generation, the reference values  $\Delta_R$ ,  $\Lambda_R$ ,  $\Xi_R$ ,  $c_{qR}$  and  $c_{pR}$  of the criteria for evaluation are each determined according to the required degree of randomness, that is, the required security against the respective cryptanalysis.

In the flowchart of Fig. 3, the order of selection of function candidates having the required resistance to the respective attacks (cryptanalyses) is not limited specifically to the order depicted in Fig. 3 but may also be changed.

25 With the random function generating method according to the present invention, it is unnecessary to select function candidates that have the required resistance to every attack shown in Fig. 3; and it also falls inside the scope of the present invention to select function candidates for at least one of higher order differential, differential-linear, partitioning and

interpolation attacks. Instead of narrowing down the function candidates one after another for a plurality of cryptanalysis methods, it is also possible to evaluate the resistance of every function candidates to the respective cryptanalyses and select functions that have the reference resistance to them.

5 While in the above the random function generating method has been described to determine parameters of composite functions each composed of two functions, it need scarcely be said that the method is similarly applicable to parameters of functions each composed of three or more functions and to the determination of parameters of one function.

10 The function randomness evaluating method and the random function generating method of the present invention, described above in the first and second embodiments, may also be prerecorded on a recording medium as programs for execution by a computer so that the programs are read out and executed by the computer to evaluate the randomness of  
15 functions and generate random functions.

#### EFFECT OF THE INVENTION

As described above, according to the present invention, in the method and apparatus for evaluating the randomness of S-box functions that  
20 serve as components of an cryptographic device or the like, there is provided, in addition to the conventional evaluating method, means for evaluating whether the functions is resistant to differential, linear, higher order differential, interpolation, partitioning and differential-linear attacks and other possible attacks, whereby it is possible to evaluate the randomness of  
25 the functions and design ciphers highly secure against the above cryptanalyses.

Furthermore, since functions each formed by a combination of a

function resistant to differential cryptanalysis and linear cryptanalysis and a function of an algebraic structure different from that of the first-mentioned function are selected as candidate functions, functions resistant not only to the differential and linear cryptanalyses but also to attacks utilizing the algebraic structure, such as high order differential and interpolation attacks can be narrowed down from a small number of candidates.

Moreover, such a procedure as depicted in Fig. 3 allows efficient narrowing down of functions with a small amount of computational complexity.

Besides, by selecting candidate functions from combinations of functions of well-known different algebraic structures instead of selecting the candidates at random, it is also easy to show that the S-box has no trap-door (a secret trick that enables only a designer to cryptanalyze the cipher concerned).

The random function thus evaluated and generated by the present invention is used as the S-box formed as by a ROM to generate an irregular outputs from the input to a cryptographic device which conceals data fast and securely.

WHAT IS CLAIMED IS:

1. A function randomness evaluating apparatus comprising at least one of:

higher-order-differential cryptanalysis resistance evaluating means for calculating the minimum value of the degree of a Boolean polynomial for input bits by which output bits of a function to be evaluated are expressed, and evaluating that the larger said minimum value, the higher the resistance of said function to higher order differential cryptanalysis is;

interpolation-cryptanalysis resistance evaluating means for: when fixing a key  $y$  and letting  $x$  denote the input of a function to be evaluated, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over Galois field which is composed of elements equal to a prime  $p$  or a power of said prime  $p$ ; calculating the number of terms of said polynomial; and evaluating the resistance of said function to interpolation cryptanalysis based on the result of said calculation;

partitioning-cryptanalysis resistance evaluating means for: dividing all inputs of a function to be evaluated and the corresponding outputs into input subsets and output subsets; calculating an imbalance of the relationship between the subset of an input and the subset of the corresponding output with respect to their average corresponding relationship; and evaluating the resistance of said function to partitioning cryptanalysis based on the result of said calculation; and

differential-linear-cryptanalysis resistance evaluating means for: calculating, for all sets of input difference  $\Delta x$  and output mask value  $\Gamma y$  of a function  $S(x)$  to be evaluated, the number of inputs  $x$  for which the inner product of  $(S(x)+S(x \Delta x))$  and said output mask value  $\Gamma y$  is 1; and

evaluating the resistance of said function to differential-linear cryptanalysis based on the result of calculation.

2. The function randomness evaluating apparatus of claim 1, wherein:

said partitioning-cryptanalysis resistance evaluating means is means for: dividing an input set F and an output set G of said function into u input subsets  $\{F_0, F_1, \dots, F_{u-1}\}$  and v output subsets  $\{G_0, G_1, \dots, G_{v-1}\}$ ; for each partition-pair  $(F_i, G_j)$  ( $i = 0, \dots, u-1$ ;  $j = 0, 1, \dots, v-1$ ), calculating the maximum one of probabilities that all outputs y corresponding to all inputs x of the input subset  $F_i$  belong to the respective output subsets  $G_j$  ( $j = 0, \dots, v-1$ ); calculating a measure  $I_s(F, G)$  of an average imbalance of a partition-pair  $(F, G)$  based on all maximum values calculated for all partition pairs; and evaluating the resistance of said function to said partitioning cryptanalysis based on said measure; and

said differential-linear cryptanalysis resistance evaluating means is means for: calculating the following equation for every set of said input difference  $\Delta x$  except 0 and said output mask value  $\Gamma y$  except 0

$$\xi_s(\Delta x, \Gamma y) = \left| 2 \times \# \{x \in GF(2)^n \mid (S(x) + S(x + \Delta x)) \bullet \Gamma y = 1\} - 2^n \right|;$$

calculating the maximum value  $\Xi$  among the calculation results; and evaluating the resistance of said function to said differential-linear cryptanalysis based on said value  $\Xi$ .

3. The function randomness evaluating apparatus of claim 1 or 2, further comprising at least one of:

differential-cryptanalysis resistance evaluating means for calculating, for a function  $S(x)$  to be evaluated, the number of inputs x that satisfy  $S(x) + S(x + S(x + \Delta x)) = \Delta y$  for every set  $(\Delta x, \Delta y)$  and evaluating the

resistance of said function to differential cryptanalysis based on the result of said calculation; and

linear-cryptanalysis resistance evaluating means for calculating, for a function to be evaluated, the number of inputs  $x$  for which the inner product of the input  $x$  and its mask value  $\Gamma x$  is equal to the inner product of a function output value  $S(x)$  and its mask value  $\Gamma y$  and evaluating the resistance of said function to linear cryptanalysis based on the result of said calculation.

4. The randomness evaluating apparatus of claim 3, wherein said linear-cryptanalysis resistance evaluating means is means for calculating the following equation

$$\lambda_s(\Gamma x, \Gamma y) = \left| 2 \times \# \{x \in (2)^n \mid x \cdot \Gamma x = S(x) \cdot \Gamma y\} - 2^n \right|$$

for every set of mask values  $(\Gamma x, \Gamma y)$  except  $\Gamma y=0$ , and evaluating the resistance of said function to said linear cryptanalysis based on a criterion defined by the following equation

$$\Lambda_s = \max \lambda_s(\Gamma x, \Gamma y).$$

5. The randomness evaluating apparatus of claim 3, wherein said differential-cryptanalysis resistance evaluating means is means for calculating the following equation

$$\delta_s(\Delta x, \Delta y) = \# \{x \in GF(2)^n \mid S(x) + S(x + \Delta x) = \Delta y\}$$

for every set of difference values  $(\Delta x, \Delta y)$  except  $\Delta x=0$ , and evaluating the resistance of said function to said differential cryptanalysis based on a criterion defined by the following equation

$$\Delta_s = \max \delta_s(\Delta x, \Delta y).$$

6. A random function generating apparatus comprising:

candidate function generating means for generating candidate



functions each formed by a combination of a plurality of functions of different algebraic structures and having a plurality of parameters;

resistance evaluating means for evaluating the resistance of each of said candidate functions to a cryptanalysis; and

selecting means for selecting those of said resistance-evaluated candidate functions which have highly resistant to said cryptanalysis.

7. The random function generating apparatus of claim 6, wherein one of said plurality of functions of different algebraic structures is resistant to each of differential cryptanalysis and linear cryptanalysis.

8. The random function generating apparatus of claim 6 or 7, wherein said resistance evaluating means comprises at least one of:

higher-order-differential cryptanalysis resistance evaluating means for: calculating the minimum value of the degree of a Boolean polynomial for input bits by which output bits of each of said candidate functions are expressed; and evaluating the resistance of said each candidate function to higher order cryptanalysis based on the result of said calculation;

interpolation-cryptanalysis resistance evaluating means for: when fixing a key  $y$  and letting  $x$  denote the input of said each candidate, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over Galois field which is composed of elements equal to a prime  $p$  or a power of said prime  $p$ ; calculating the number of terms of said polynomial; and evaluating the resistance of said each candidate function to interpolation cryptanalysis based on the result of said calculation;

partitioning-cryptanalysis resistance evaluating means for: dividing all inputs of a function to be evaluated and the corresponding outputs into input subsets and output subsets; calculating an imbalance of the relationship between the subset of an input and the subset of the

corresponding output with respect to their average corresponding relationship; and evaluating the resistance of said function to partitioning cryptanalysis based on the result of said calculation; and

differential-linear cryptanalysis resistance evaluating means for: calculating, for every set of input difference  $\Delta x$  and output mask value  $\Gamma y$  of a function  $S(x)$  to be evaluated, the number of inputs  $x$  for which the inner product of  $(S(x)+S(x\Delta x))$  and said output mask value  $\Gamma y$  is 1; and evaluating the resistance of said function to differential-linear cryptanalysis based on the result of said calculation.

9. A method for evaluating the randomness of the input/output relationship of a function, said method comprising at least one of:

(a) a higher-order-differential cryptanalysis resistance evaluating step of: letting said function be represented by  $S(x)$ , calculating the minimum value of the degree of a Boolean polynomial for input bits of said function  $S(x)$  by which its output bits are expressed; and evaluating the resistance of said function to higher order cryptanalysis based on the result of said calculation;

(b) a differential-linear cryptanalysis resistance evaluating step of: calculating, for every set of input difference  $\Delta x$  and output mask value  $\Gamma y$  of a function  $S(x)$  to be evaluated, the number of inputs  $x$  for which the inner product of  $(S(x)+S(x\Delta x))$  and said output mask value  $\Gamma y$  is 1; and evaluating the resistance of said function to differential-linear cryptanalysis based on the result of said calculation;

(c) a partitioning-cryptanalysis resistance evaluating step of: dividing all inputs of a function to be evaluated and the corresponding outputs into input subsets and output subsets; calculating an imbalance of the relationship between the subset of an input and the subset of the

corresponding output with respect to their average corresponding relationship; and evaluating the resistance of said function to partitioning cryptanalysis based on the result of said calculation; and

(d) an interpolation-cryptanalysis resistance evaluating step of: when fixing a key  $y$  and letting  $x$  denote the input of said each candidate, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over Galois field which is composed of elements equal to a prime  $p$  or a power of said prime  $p$ ; calculating the number of terms of said polynomial; and evaluating the resistance of said function to interpolation cryptanalysis.

10. The randomness evaluating method of claim 9, wherein:

said differential-linear cryptanalysis resistance evaluating step (b) is a step of: letting the input difference and output mask value of said function  $S(x)$  be representing by  $\Delta x$  and  $\Gamma y$ , respectively, calculating the following equation for every set of said input difference  $\Delta x$  except 0 and said output mask value  $\Gamma y$  except 0

$$\xi_s(\Delta x, \Gamma y) = \left| 2 \times \# \{x \in GF(2)^n \mid (S(x) + S(x + \Delta x) \bullet \Gamma y = 1)\} - 2^n \right|;$$

calculating the maximum value  $\Xi$  among the calculation results; and evaluating the resistance of said function to said differential-linear cryptanalysis using said value  $\Xi$ ; and

said partitioning-cryptanalysis resistance evaluating step (c) is a step of: dividing an input set  $F$  and an output set  $G$  of said function into  $u$  input subsets  $\{F_0, F_1, \dots, F_{u-1}\}$  and  $v$  output subsets  $\{G_0, G_1, \dots, G_{v-1}\}$ ; for each partition-pair  $(F_i, G_j)$  ( $i = 0, \dots, u-1; j = 0, 1, \dots, v-1$ ), calculating the maximum one of probabilities that all outputs  $y$  corresponding to all inputs  $x$  of the input subset  $F_i$  belong to the respective output subsets  $G_j$  ( $j = 0, \dots, v-1$ ); calculating a measure  $I_s(F, G)$  of an average imbalance of a partition-pair

(F, G) based on all maximum values calculated for all partition pairs; and evaluating the resistance of said function to said partitioning cryptanalysis based on said measure.

11. The randomness evaluating method of claim 9 or 10, further comprising at least one of:

(e) a differential-cryptanalysis resistance evaluating step of: letting the output difference value of said function  $S(x)$  be represented by  $\Delta x$ , calculating the number of inputs  $x$  that satisfy  $S(x)+S(x+S(x+\Delta x))=\Delta y$  for every set  $(\Delta x, \Delta y)$  except  $\Delta x=0$ ; and evaluating the resistance of said function to differential cryptanalysis based on the result of said calculation; and

(f) a linear-cryptanalysis resistance evaluating means for calculating, for said function  $S(x)$ , the number of inputs  $x$  for which the inner product of the input  $x$  and its mask value  $\Gamma x$  is equal to the inner product of a function output value  $S(x)$  and its mask value  $\Gamma y$  and evaluating the resistance of said function to linear cryptanalysis based on the result of said calculation.

12. The randomness evaluating method of claim 11, wherein, letting the number of bits of said input  $x$  be represented by  $n$ :

said differential-cryptanalysis resistance evaluating step (e) is a step of: calculating the following equation

$$\delta_s(\Delta x, \Delta y) = \# \{x \in GF(2)^n \mid S(x) + S(x + \Delta x) = \Delta y\}$$

for every set of difference values  $(\Delta x, \Delta y)$  except  $\Delta x=0$ ; and evaluating the resistance of said function to said differential cryptanalysis based on a criterion defined by the following equation

$$\Delta_s = \max \delta_s(\Delta x, \Delta y); \text{ and}$$

said linear-cryptanalysis resistance evaluating step (f) is a step of: letting the mask value of said input  $x$  be represented by  $\Gamma x$ , calculating the

following equation

$$\lambda_s(\Gamma x, \Gamma y) = \left| 2 \times \# \{x \in (2)^n \mid x \bullet \Gamma x = S(x) \bullet \Gamma y\} - 2^n \right|$$

for every set of mask values ( $\Gamma x$ ,  $\Gamma y$ ) except  $\Gamma y=0$ ; and evaluating the resistance of said function to said linear cryptanalysis based on a criterion defined by the following equation

$$\Lambda_s = \max \lambda_s(\Gamma x, \Gamma y).$$

13. A random function generating method comprising the steps of:

- (a) setting various values as each parameter for candidate functions and calculating output values corresponding to various input values;
- (b) storing the results of said calculation in storage means; and
- (c) evaluating the resistance of each of said candidate functions to a cryptanalysis based on values stored in said storage means, and selectively outputting candidate function highly resistant to said cryptanalysis; and

wherein said step (c) comprising at least one of:

- (c-1) a higher-order cryptanalysis resistance evaluating step of: calculating the minimum value of the degree of a Boolean polynomial for input bits of each of said candidate functions by which its output bits are expressed; evaluating the resistance of said each candidate function to higher order cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined first reference and discarding the others;
- (c-2) a differential-linear cryptanalysis resistance evaluating step of: calculating, for every set of input difference  $\Delta x$  and output mask value  $\Gamma y$  of each candidate function  $S(x)$ , the number of inputs  $x$  for which the inner product of  $(S(x)+S(x \Delta x))$  and said output mask value  $\Gamma y$  is 1; evaluating the resistance of said function to differential-linear cryptanalysis based on

the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined second reference and discarding the others;

(c-3) a partitioning-cryptanalysis resistance evaluating step of: dividing all inputs of each candidate function and the corresponding outputs into input subsets and output subsets; calculating an imbalance of the relationship between the subset of an input and the subset of the corresponding output with respect to their average corresponding relationship; evaluating the resistance of said each candidate function to said partitioning cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined third reference and discarding the others; and

(c-4) an interpolation-cryptanalysis resistance evaluating step of: when fixing a key  $y$  and letting  $x$  denote the input of said each candidate, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over Galois field which is composed of elements equal to a prime  $p$  or a power of said prime  $p$ ; calculating the number of terms of said polynomial; evaluating the resistance of said function to interpolation cryptanalysis; and leaving those of said candidate functions whose resistance is higher than a predetermined fourth reference and discarding the others.

14. The random function generating apparatus of claim 13, wherein: said differential-linear-cryptanalysis resistance evaluating step (c-2) includes a step of: letting the output mask value be represented by  $\Gamma y$ , calculating the following equation for every set of said input difference  $\Delta x$  except 0 and said output mask value  $\Gamma y$  except 0

$$\xi_s(\Delta x, \Gamma y) = \left| 2 \times \# \{x \in GF(2)^n \mid (S(x) + S(x + \Delta x)) \bullet \Gamma y = 1\} - 2^n \right|;$$

calculating the maximum value  $\Xi$  among the calculation results; and evaluating the resistance of said candidate function to said differential-linear cryptanalysis based on said value  $\Xi$ ; and said partitioning cryptanalysis resistance evaluating step (3) includes a step of dividing an input set F and an output set G of said function into u input subsets  $\{F_0, F_1, \dots, F_{u-1}\}$  and v output subsets  $\{G_0, G_1, \dots, G_{v-1}\}$ ; for each partition-pair  $(F_i, G_j)$  ( $i = 0, \dots, u-1; j = 0, 1, \dots, v-1$ ), calculating the maximum one of probabilities that all outputs y corresponding to all inputs x of the input subset  $F_i$  belong to the respective output subsets  $G_j$  ( $j = 0, \dots, v-1$ ); calculating a measure  $I_s(F, G)$  of an average imbalance of a partition-pair  $(F, G)$  based on all maximum values calculated for all partition pairs; and evaluating the resistance of said candidate function to said partitioning cryptanalysis based on said measure.

15. The random function generating method of claim 13 or 14, wherein:

said step (c-1) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said first reference by a first predetermined width, and executing again the evaluation and selecting process;

said step (c-2) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said second reference by a second predetermined width, and executing again the evaluation and selecting process;

said step (c-3) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said third reference by a third predetermined width, and executing again the evaluation and selecting process; and

said step (c-4) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said fourth reference by a fourth predetermined width, and executing again the evaluation and selecting process.

16. The random function generating method of claim 13 or 14, further comprising at least one of:

(c-5) a differential-cryptanalysis resistance evaluating step of: calculating, for each candidate function  $S(x)$ , the number of inputs  $x$  that satisfy  $S(x) + S(x + \Delta x) = \Delta y$  for every set  $(\Delta x, \Delta y)$  except  $\Delta x$ ; evaluating the resistance of said each candidate function to differential cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined fifth reference and discarding the others; and

(c-6) a linear-cryptanalysis resistance evaluating step of: calculating, for each candidate function, the number of inputs  $x$  for which the inner product of the input  $x$  and its mask value  $\Gamma x$  is equal to the inner product of a function output value  $S(x)$  and its mask value  $\Gamma y$ ; evaluating the resistance of said each candidate function to linear cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined sixth reference and discarding the others.

17. The random function generating method of claim 16, wherein: said differential-cryptanalysis resistance evaluating step (c-5) includes a step of: calculating the following equation

$$\delta_s(\Delta x, \Delta y) = \# \{x \in GF(2)^n \mid S(x) + S(x + \Delta x) = \Delta y\}$$

for every set of difference values  $(\Delta x, \Delta y)$  except  $\Gamma y = 0$ ; and evaluating the resistance of said each candidate function to said differential



cryptanalysis based on a criterion defined by the following equation

$$\Delta_s = \max \delta_s (\Delta x, \Delta y); \text{ and}$$

said linear-cryptanalysis resistance evaluating step (c-6) includes a step of: calculating the following equation

$$\lambda_s (\Gamma x, \Gamma y) = \left| 2 \times \# \left\{ x \in (2)^n \mid x \bullet \Gamma x = S(x) \bullet \Gamma y \right\} - 2^n \right|$$

for every set of mask values ( $\Gamma x$ ,  $\Gamma y$ ); and evaluating the resistance of said each candidate function to said linear cryptanalysis based on a criterion defined by the following equation

$$\Lambda_s = \max \lambda_s (\Gamma x, \Gamma y).$$

18. The random function generating method of claim 16 or 17,

wherein:

said step (c-5) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said fifth reference by a fifth predetermined width, and executing again the evaluation and selecting process; and

said step (c-6) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said sixth reference by a sixth predetermined width, and executing again the evaluation and selecting process.

19. The random function generating method of claim 13, 14, or 15, wherein said candidate functions are each a composite function composed of at least one function resistant to said differential cryptanalysis and said linear cryptanalysis and at least one function of an algebraic structure different from that of said at least one function.

20. A recording medium having recorded thereon a random function generating method as a computer program, said program comprising the

steps of:

- (a) setting various values as each parameter for candidate functions and calculating output values corresponding to various input values;
- (b) storing the results of said calculation in storage means; and
- (c) evaluating the resistance of each of said candidate functions to a cryptanalysis based on values stored in said storage means, and selectively outputting candidate function highly resistant to said cryptanalysis; and

wherein said step (c) comprising at least one of:

(c-1) a higher-order cryptanalysis resistance evaluating step of: calculating the minimum value of the degree of a Boolean polynomial for input bits of each of said candidate functions by which its output bits are expressed; evaluating the resistance of said each candidate function to higher order cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined first reference and discarding the others;

(c-2) a differential-linear cryptanalysis resistance evaluating step of: calculating, for every set of input difference  $\Delta x$  and output mask value  $\Gamma y$  of each candidate function  $S(x)$ , the number of inputs  $x$  for which the inner product of  $(S(x)+S(x\Delta x))$  and said output mask value  $\Gamma y$  is 1; evaluating the resistance of said function to differential-linear cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined second reference and discarding the others;

(c-3) a partitioning-cryptanalysis resistance evaluating step of: dividing all inputs of each candidate function and the corresponding outputs into input subsets and output subsets; calculating an imbalance of the relationship between the subset of an input and the subset of the

corresponding output with respect to their average corresponding relationship; evaluating the resistance of said each candidate function to said partitioning cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined third reference and discarding the others; and

(c-4) an interpolation-cryptanalysis resistance evaluating step of: when fixing a key  $y$  and letting  $x$  denote the input of said each candidate, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over Galois field which is composed of elements equal to a prime  $p$  or a power of said prime  $p$ ; calculating the number of terms of said polynomial; evaluating the resistance of said function to interpolation cryptanalysis; and leaving those of said candidate functions whose resistance is higher than a predetermined fourth reference and discarding the others.

21. The recording medium of claim 20, wherein:

said differential-linear-cryptanalysis resistance evaluating step (c-2) includes a step of: letting the output mask value be represented by  $\Gamma y$ , calculating the following equation for every set of said input difference  $\Delta x$  except 0 and said output mask value  $\Gamma y$  except 0

$$\xi_s(\Delta x, \Gamma y) = \left| 2 \times \# \{x \in GF(2)^n \mid (S(x) + S(x + \Delta x)) \bullet \Gamma y = 1\} - 2^n \right|;$$

calculating the maximum value  $\Xi$  among the calculation results; and evaluating the resistance of said candidate function to said differential-linear cryptanalysis based on said value  $\Xi$ ; and

said partitioning cryptanalysis resistance evaluating step (3) includes a step of dividing an input set  $F$  and an output set  $G$  of said function into  $u$  input subsets  $\{F_0, F_1, \dots, F_{u-1}\}$  and  $v$  output subsets  $\{G_0, G_1, \dots, G_{v-1}\}$ ; for each partition-pair  $(F_i, G_j)$  ( $i = 0, \dots, u-1; j = 0, 1, \dots, v-1$ ), calculating the

maximum one of probabilities that all outputs  $y$  corresponding to all inputs  $x$  of the input subset  $F_i$  belong to the respective output subsets  $G_j$  ( $j = 0, \dots, v-1$ ); calculating a measure  $I_S(F, G)$  of an average imbalance of a partition-pair  $(F, G)$  based on all maximum values calculated for all partition pairs; and evaluating the resistance of said candidate function to said partitioning cryptanalysis based on said measure.

22. The recording medium of claim 20 or 21, wherein:

said step (c-1) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said first reference by a first predetermined width, and executing again the evaluation and selecting process;

said step (c-2) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said second reference by a second predetermined width, and executing again the evaluation and selecting process;

said step (c-3) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said third reference by a third predetermined width, and executing again the evaluation and selecting process; and

said step (c-4) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said fourth reference by a fourth predetermined width, and executing again the evaluation and selecting process.

23. The recording medium of claim 20 or 21, wherein said program includes at least one of:

(c-5) a differential-cryptanalysis resistance evaluating step of: calculating, for each candidate function  $S(x)$ , the number of inputs  $x$  that

satisfy  $S(x)+S(x+S(x+\Delta x))=\Delta y$  for every set  $(\Delta x, \Delta y)$  except  $\Delta x$ ; evaluating the resistance of said each candidate function to differential cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined fifth reference and discarding the others; and

(c-6) a linear-cryptanalysis resistance evaluating step of: calculating, for each candidate function, the number of inputs  $x$  for which the inner product of the input  $x$  and its mask value  $\Gamma x$  is equal to the inner product of a function output value  $S(x)$  and its mask value  $\Gamma y$ ; evaluating the resistance of said each candidate function to linear cryptanalysis based on the result of said calculation; and leaving those of said candidate functions whose resistance is higher than a predetermined sixth reference and discarding the others.

24. The recording medium of claim 23, wherein:

said differential-cryptanalysis resistance evaluating step (c-5) includes a step of: calculating the following equation

$$\delta_s(\Delta x, \Delta y) = \#\{x \in GF(2)^n \mid S(x) + S(x + \Delta x) = \Delta y\}$$

for every set of difference values  $(\Delta x, \Delta y)$  except  $\Gamma y=0$ ; and evaluating the resistance of said each candidate function to said differential cryptanalysis based on a criterion defined by the following equation

$$\Delta_s = \max \delta_s(\Delta x, \Delta y); \text{ and}$$

said linear-cryptanalysis resistance evaluating step (c-6) includes a step of: calculating the following equation

$$\lambda_s(\Gamma x, \Gamma y) = \left| 2 \times \#\{x \in (2)^n \mid x \bullet \Gamma x = S(x) \bullet \Gamma y\} - 2^n \right|$$

for every set of mask values  $(\Gamma x, \Gamma y)$ ; and evaluating the resistance of said each candidate function to said linear cryptanalysis based on a criterion

defined by the following equation

$$\Lambda_s = \max \lambda_s(\Gamma x, \Gamma y).$$

25. The recording medium of claim 23 or 24, wherein:

said step (c-5) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said fifth reference by a fifth predetermined width, and executing again the evaluation and selecting process; and

said step (c-6) includes a step of: when no candidate function remains undiscarded, easing the candidate function selecting condition by changing said sixth reference by a sixth predetermined width, and executing again the evaluation and selecting process.

26. The recording medium of claim 20, 21, or 22, wherein said candidate functions are each a composite function composed of at least one function resistant to said differential cryptanalysis and said linear cryptanalysis and at least one function of an algebraic structure different from that of said at least one function.

27. A recording medium having recorded thereon as a program a method for evaluating the randomness of the input/output relationship of a function, said program comprising at least one of:

(a) a higher-order-differential cryptanalysis resistance evaluating step of: letting said function be represented by  $S(x)$ , calculating the minimum value of the degree of a Boolean polynomial for input bits of said function  $S(x)$  by which its output bits are expressed; and evaluating the resistance of said function to higher order cryptanalysis based on the result of said calculation;

(b) a differential-linear cryptanalysis resistance evaluating step of: calculating, for every set of input difference  $\Delta x$  and output mask value  $\Gamma y$

of a function  $S(x)$  to be evaluated, the number of inputs  $x$  for which the inner product of  $(S(x)+S(x \Delta x))$  and said output mask value  $\Gamma y$  is 1; and evaluating the resistance of said function to differential-linear cryptanalysis based on the result of said calculation;

(c) a partitioning-cryptanalysis resistance evaluating step of: dividing all inputs of a function to be evaluated and the corresponding outputs into input subsets and output subsets; calculating an imbalance of the relationship between the subset of an input and the subset of the corresponding output with respect to their average corresponding relationship; and evaluating the resistance of said function to partitioning cryptanalysis based on the result of said calculation; and

(d) an interpolation-cryptanalysis resistance evaluating step of: when fixing a key  $y$  and letting  $x$  denote the input of said each candidate, expressing an output  $y$  by  $y = f_k(x)$  using a polynomial over Galois field which is composed of elements equal to a prime  $p$  or a power of said prime  $p$ ; calculating the number of terms of said polynomial; and evaluating the resistance of said function to interpolation cryptanalysis.

28. The recording medium of claim 27, wherein:

said differential-linear cryptanalysis resistance evaluating step (b) is a step of: letting the input difference and output mask value of said function  $S(x)$  be representing by  $\Delta x$  and  $\Gamma y$ , respectively, calculating the following equation for every set of said input difference  $\Delta x$  except 0 and said output mask value  $\Gamma y$  except 0

$$\xi_s(\Delta x, \Gamma y) = \left| 2 \times \# \{x \in GF(2)^n \mid (S(x) + S(x + \Delta x)) \bullet \Gamma y = 1\} - 2^n \right|;$$

calculating the maximum value  $\Xi$  among the calculation results; and evaluating the resistance of said function to said differential-linear

cryptanalysis using said value  $\Xi$ ; and

said partitioning-cryptanalysis resistance evaluating step (c) is a step of: dividing an input set F and an output set G of said function into u input subsets  $\{F_0, F_1, \dots, F_{u-1}\}$  and v output subsets  $\{G_0, G_1, \dots, G_{v-1}\}$ ; for each partition-pair  $(F_i, G_j)$  ( $i = 0, \dots, u-1; j = 0, 1, \dots, v-1$ ), calculating the maximum one of probabilities that all outputs y corresponding to all inputs x of the input subset  $F_i$  belong to the respective output subsets  $G_j$  ( $j = 0, \dots, v-1$ ); calculating a measure  $I_s(F, G)$  of an average imbalance of a partition-pair  $(F, G)$  based on all maximum values calculated for all partition pairs; and evaluating the resistance of said function to said partitioning cryptanalysis based on said measure.

29. The recording medium of claim 27 or 28, said program further comprising at least one of:

(e) a differential-cryptanalysis resistance evaluating step of: letting the output difference value of said function  $S(x)$  be represented by  $\Delta x$ , calculating the number of inputs x that satisfy  $S(x) \oplus S(x + \Delta x) = \Delta y$  for every set  $(\Delta x, \Delta y)$  except  $\Delta x = 0$ ; and evaluating the resistance of said function to differential cryptanalysis based on the result of said calculation; and

(f) a linear-cryptanalysis resistance evaluating means for calculating, for said function  $S(x)$ , the number of inputs x for which the inner product of the input x and its mask value  $\Gamma x$  is equal to the inner product of a function output value  $S(x)$  and its mask value  $\Gamma y$  and evaluating the resistance of said function to linear cryptanalysis based on the result of said calculation.

30. The recording medium of claim 29, wherein, letting the number of bits of said input x be represented by n:

said differential-cryptanalysis resistance evaluating step (e) is a step



of: calculating the following equation

$$\delta_s(\Delta x, \Delta y) = \# \{x \in GF(2)^n \mid S(x) + S(x + \Delta x) = \Delta y\}$$

for every set of difference values  $(\Delta x, \Delta y)$  except  $\Delta x=0$ ; and evaluating the resistance of said function to said differential cryptanalysis based on a criterion defined by the following equation

$$\Delta_s = \max \delta_s(\Delta x, \Delta y); \text{ and}$$

said linear-cryptanalysis resistance evaluating step (f) is a step of: letting the mask value of said input  $x$  be represented by  $\Gamma x$ , calculating the following equation

$$\lambda_s(\Gamma x, \Gamma y) = \left| 2 \times \# \left\{ x \in (2)^n \mid x \bullet \Gamma x = S(x) \bullet \Gamma y \right\} - 2^n \right|$$

for every set of mask values  $(\Gamma x, \Gamma y)$  except  $\Gamma y=0$ ; and evaluating the resistance of said function to said linear cryptanalysis based on a criterion defined by the following equation

$$\Lambda_s = \max \lambda_s(\Gamma x, \Gamma y).$$

## ABSTRACT

In the evaluation of the randomness of an S-box, measures of resistance to higher order cryptanalysis, interpolation cryptanalysis, partitioning cryptanalysis and differential-linear cryptanalysis and necessary conditions for those measures to have resistance to each cryptanalysis are set, then for functions as candidates for the S-box, it is evaluated whether one or all of the conditions are satisfied, and those of the candidate functions for which one or all of the conditions are satisfied are selected as required. It is also possible to further evaluate the resistance of such selected functions to at least one of differential cryptanalysis and linear cryptanalysis and select those of the candidate functions which are resistant to at least one of the cryptanalyses as required.

03463907-020200

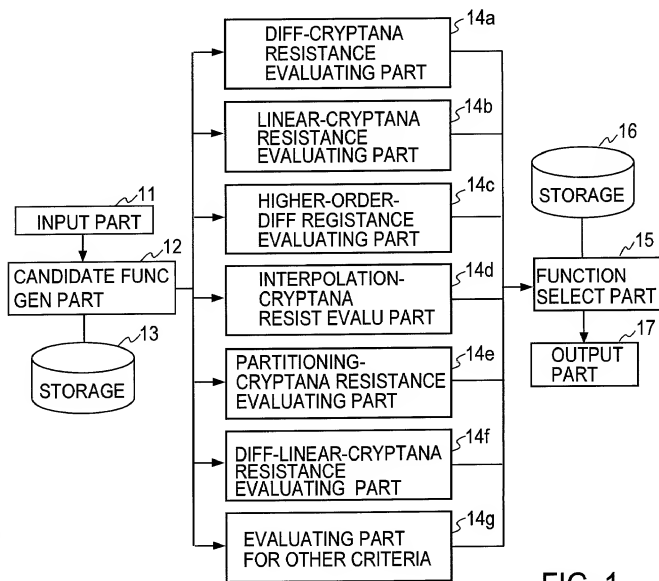


FIG. 1

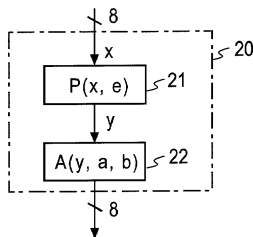
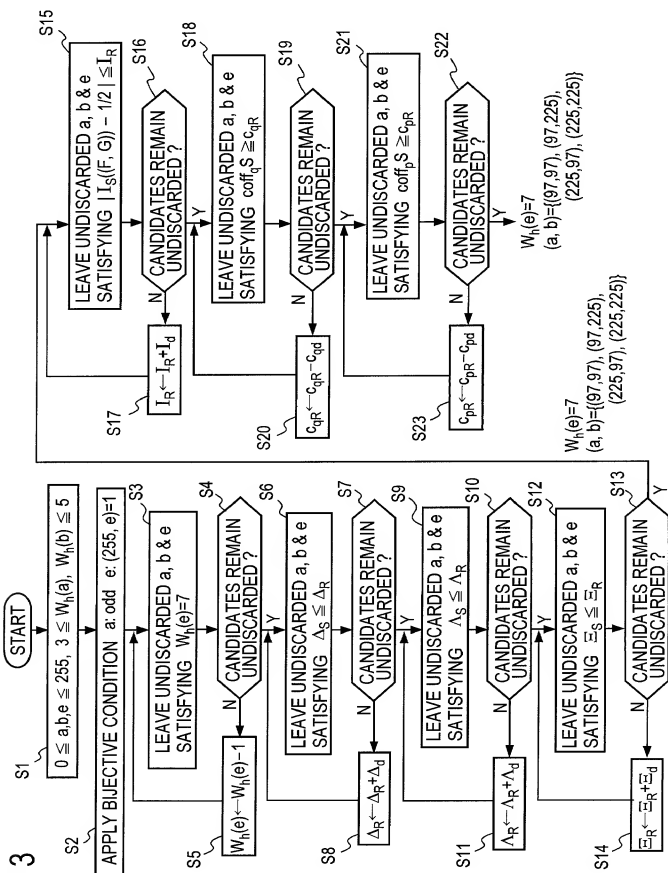


FIG. 2

S1 START



# DECLARATION FOR PATENT APPLICATION

As a below-named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name.

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled:

Please see attached sheet

the specification of which: (check one)

[XX] is attached hereto. ☒ was filed on 01/06/99 as United States Patent Application Serial No. or PCT International Application Number PCT/JP99/02924, and was amended on 19 (if applicable).

I hereby state that I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above.

I acknowledge the duty to disclose information which is material to the patentability of this application in accordance with 37 CFR § 1.56(a).

Prior Foreign Application(s): I hereby claim foreign priority benefits under 35 U.S.C. § 119(a)-(d) or § 365(b) of any foreign application(s) for patent or inventor's certificate listed below, or § 365(a) of any PCT international application which designated at least one country other than the United States of America, listed below and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on which priority is claimed:

153066/98

(Application No.)

Japan

(Country)

02/06/98

(Day/Month/Year Filed)

Priority Claimed

☒ [X] [ ]

Yes No

[ ] [ ]

Yes No

[ ] [ ]

Yes No

(Application No.)

(Country)

(Day/Month/Year Filed)

(Application No.)

(Country)

(Day/Month/Year Filed)

I hereby claim the benefit under Title 35, United States Code § 119(e) of any United States provisional application(s) listed below:

Application No.

Filing Date

I hereby claim the benefit under 35 U.S.C. § 120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by 35 U.S.C. § 112, first paragraph, I acknowledge the duty to disclose material information as defined in 37 CFR § 1.56(a) which occurred between the filing date of the prior application and the national or PCT international filing date of this application:

(U.S. Application Serial No.)

(U.S. Filing Date)

(Status--patented, pending, abandoned)

(U.S. Application Serial No.)

(U.S. Filing Date)

(Status--patented, pending, abandoned)

I hereby appoint Elliott I. Pollock, Registration No. 16,906; George Vande Sande, Registration No. 17,276; Burton A. Amernick, Registration No. 24,852; Stanley B. Green, Registration No. 24,351; Richard Wiener, Registration No. 18,741; Townsend M. Belser, Jr., Registration No. 22,956; Morris Liss, Registration No. 24,510; Martin Abramson, Registration No. 25,787; George R. Pettit, Registration No. 27,369; Elzbieta Chlopiecka, Registration No. 32,267; Eric J. Franklin, Registration No. 37,134; and Jeffri A. Kaminski, Reg. No. 42,709, my attorneys with full power of substitution and revocation, to prosecute this application and to transact all business in the Patent and Trademark Office connected therewith.

Send Correspondence and Direct Telephone Calls to:

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I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements are made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under 18 U.S.C. § 1001, and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

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Inventor's Signature: Shiho Mortai

Date January 18, 2000

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Japan

# DECLARATION FOR PATENT APPLICATION

Page Two

2-D  
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2-D  
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7-D  
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5-D  
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Full name of eighth joint inventor (if any): \_\_\_\_\_  
Inventor's Signature \_\_\_\_\_ Date \_\_\_\_\_  
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TITLE OF INVENTION: "APPARATUS AND METHOD FOR EVALUATING RANDOMNESS OF  
FUNCTIONS, RANDOM FUNCTION GENERATING APPARATUS AND  
METHOD, AND RECORDING MEDIUM HAVING RECORDED THEREON  
PROGRAMS FOR IMPLEMENTING THE METHODS"

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